

SIMULATION OF FUNCTIONALLY GRADED MATERIAL PLATE SUBJECT TO THERMO MECHANICAL LOADS

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Abstract

Functionally gradient materials (FGM) widely used materials as they have more advantages as compared to traditional composites. The objective of this paper is to simulate and analyse an FGM plate subjected to a mechanical load under thermal environment. The properties are assumed to vary according to Power law distribution in terms of the volume fractions of the constituents. Numerical analysis using finite element method is followed by simulation in ANSYS software. The displacement fields for simply supported FGM plate under thermomechanical loads are analysed.

Keywords:

ANSYS;

FEM;

FGM;

Plate;

Thermo-mechanical.

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1. Introduction

The material property of the FGM can be used to accomplish demands in engineering utilizations to get the advantage of the properties of individual material. This is because of the material composition of the FGM changes in a preferred direction. FGMs are useful in different engineering applications which include design of aerospace structures, heat engine components and nuclear power plants etc. A number of published literature has been observed for evaluation of thermos-mechanical behavior of functionally gradient material plate using finite element techniques. The main objective of the FEM-based design of heterogeneous objects is to simultaneously optimize both geometry and material distribution over the design domain. Praveen and Reddy[1] developed finite element model and employed four-noded rectangular isoparametric element having five degrees of freedom. They chose a regular mesh of 8 x 8 linear elements for the convergence studies. Reddy[2] developed linear and nonlinear finite element model and Navier solutions that covered thermos-mechanical loading. Cheng and Batra [3] found deformations due to the temperature variation in the thickness direction analytically under mechanical load by the method of asymptotic expansion. Senthil and Batra[4] used the Laplace transformation method to reduce equations to an ordinary differential equation (ODE) in the thickness direction. Qian and Batra[5] used the mesh less local Petrov–Galerkin method (MLPG) method to evaluate different integrals. Dai et.al. [6] used the element-free Galerkin method to find shape functions using the moving least squares (MLS) method. Shin [7] introduced a method for FEA-based design of FGM and optimized both geometry and material distribution over the design domain. Wang and Qin[8] developed fundamental solutions method to simulate the thermal stress distribution in (2D) functionally graded materials (FGMs). Alieldin et.al. [9] derived displacements and rotations at a point infinite element in terms of nodes of the element. Kyung and Kim[10] reported stress analysis of FG composite plates using finite element method. Xuan et.al.[11] presented an improved finite element approach with a node-based strain smoothing triangular plate elements. Alshorbagy et.al. [12] used FEM to investigate the thermoplastic behavior of a FG plate. The varying nature of FGMs makes design and analysis complicated compared to traditional materials. A functionally graded (FG) two-phase plate is analyzed using classical lamination theory, wherein each layer is given a different volume fraction. In this analysis technique properties are employed for each layer. The availability of FEA codes (e.g., ABAQUS, ANSYS, and NASTRAN) makes FEA attractive for design and analysis of FGMs.

ANSYS is a finite-element modeling package for numerically solving a wide variety of problems. These problems include static, dynamic and structural analysis problems. Craig et.al.[13] developed a new software package called Higher-Order Theory – Structural/Micro Analysis Code (HOT-SMAC) as a tool for design and analysis of FGM. Qian et.al.[14] developed a code to determine static deformations of a FG thick plate. Chi and Chung[15][16] evaluated numerically from theoretical formulations and calculated by FEM using MARC program. Shin [7] analyzed the FE models using ANSYS commercial software (ANSYS Inc.) and performed a linear elastic analysis in order to evaluate thermally independent effective properties for each ceramic volume fraction. Reddy[17] used the material property gradation through the thickness and the profile for volume fraction using Power law distribution. The researchers have used many boundary conditions which include simply supported, clamped, free and combinations. Praveen and Reddy[1] analyzed a plate which was simply supported at all edges. Cheng and Batra [3] analyzed thermo-mechanical deformations of an elastic FG elliptic plate with rigidly clamped edges. Reddy[17] employed a plate simply supported on all its edges. Senthil and Batra[4] employed rectangular simply supported FG plate with uniform temperature applied at the edges. Qian et.al. [14] used rectangular FG plate with edges held at a uniform temperature and simply supported at its edges. Ferreira et.al.[18] worked upon FG plate simply-supported at all its edges. Dai et.al.[6] analyzed the plate under the mechanical loading as well as thermal gradient.

The thermo-mechanical deformation of FGM structures have attracted the attention of many researchers in the past few years. The finite element method in conjunction with simulation software like ANSYS have proved to be very effective in calculating behavior of FG plates under various loads. In the present paper an attempt has been made to simulate FG plate using ANSYS and parametric behavior has been presented in terms of non-dimensional parameters e.g. deflection, stress etc.

2. Problem formulation

Thermo-mechanical analysis of FGM plate. The work includes to present parametric behavior of FG plate under thermomechanical load in terms of non-dimensional parameters e.g. deflection, stress and strain for various volume fraction exponents. The FG plate is subjected to simply supported boundary condition. The thermo-mechanical analysis of FGM plate is conducted using

finite element model and ANSYS software is being used for computing the response. The FGM plate is made of Aluminum and Zirconia. The physical properties of Aluminum and Zirconia are listed in Table 1.

Table 1: Physical Properties of Aluminum and Zirconia

S.No.	Property	Aluminum	Zirconia
1	Young's modulus	70 GPa	151 GPa
2	Poisson's ratio	0.3	0.3
3	Thermal conductivity	204 W/mK	2.09 W/mK
4	Coefficient of thermal expansion	$23 \times 10^{-6} / ^\circ\text{C}$	$10 \times 10^{-6} / ^\circ\text{C}$
5	Density	3000 (Kg/m^3)	3000 (Kg/m^3)

The properties of the material is assumed to follow Power law distribution. The material properties are dependent on the volume fraction V_f which follows Eq. (1):

$$V_f = (z/h+1/2)^n \quad \dots(1)$$

Where h is the thickness of the plate, z is thickness coordinate and n is the exponent.

3. Modeling and simulation

ANSYS offers a wide range of capabilities in any processor. ANSYS proves to be useful for design optimization, adaptive meshing, and customization for change of material properties, change in loading conditions, and change of element type used in any analysis.

3.1 Idealizations:

Displacement in the X, Y and Z directions are denoted by UX, UY and UZ respectively and rotations in the X, Y and Z directions are denoted by ROTX, ROTY and ROTZ respectively. The numerical model is broken up into number of “layers” in order to capture the change in properties as the material properties of the FGM change throughout the thickness, The “layers” occupy a finite portion of thickness and are treated as isotropic materials. The associated properties are layered together to establish the through-the-thickness variation of material properties. Although the layered structure does not show gradual change in material properties, yet a sufficient number of “layers” can approximate the material gradation. ANSYS offers a number of elements to choose from for the modeling of layered materials. The FGM layered plate is modeled using suitable element found from finite element model.

3.2 Model generation

Having defined material and material properties, the rectangular plate is modeled and meshed with predefined mesh size using the mesh tool. Fig.1 and 2 show the isometric views of the meshed square plate. The layers are also visible in the Fig.1 and Fig.2. The plate modeled in the present work is subjected to simply supported boundary condition i.e. along the X direction, $U_X=U_Z=0$ and along the Y direction $U_Y=U_Z=0$. It is shown in Fig.3. Having meshed the plate and applied the desired boundary conditions, a mechanical uniformly distributed load and thermal environment is applied. A command SOLVE is executed to obtain the results e.g. deflection, stress etc. Fig. 4,5 and 6 show various result parameters which includes stress, shear stress and deflection, Fig. 4,5 and 6 show the distribution of the parameters in the plate. The distribution intensity is shown with the colour variation.

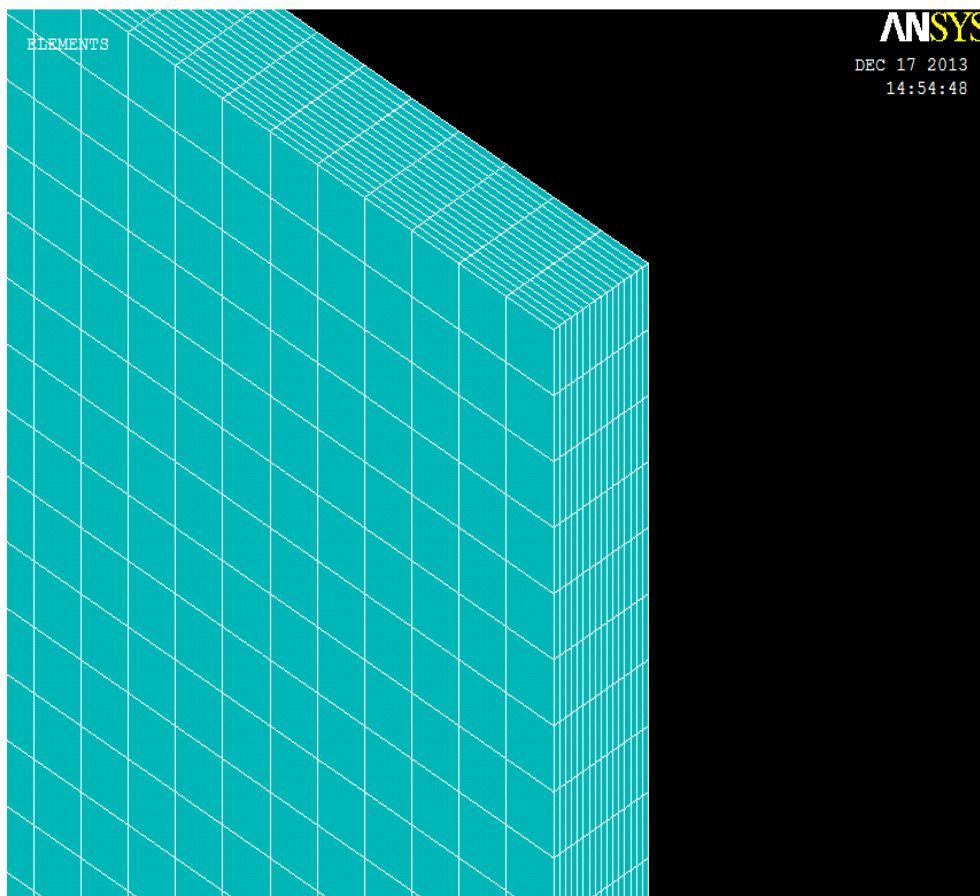


Fig. 1. Isometric zoomed view of meshed plate

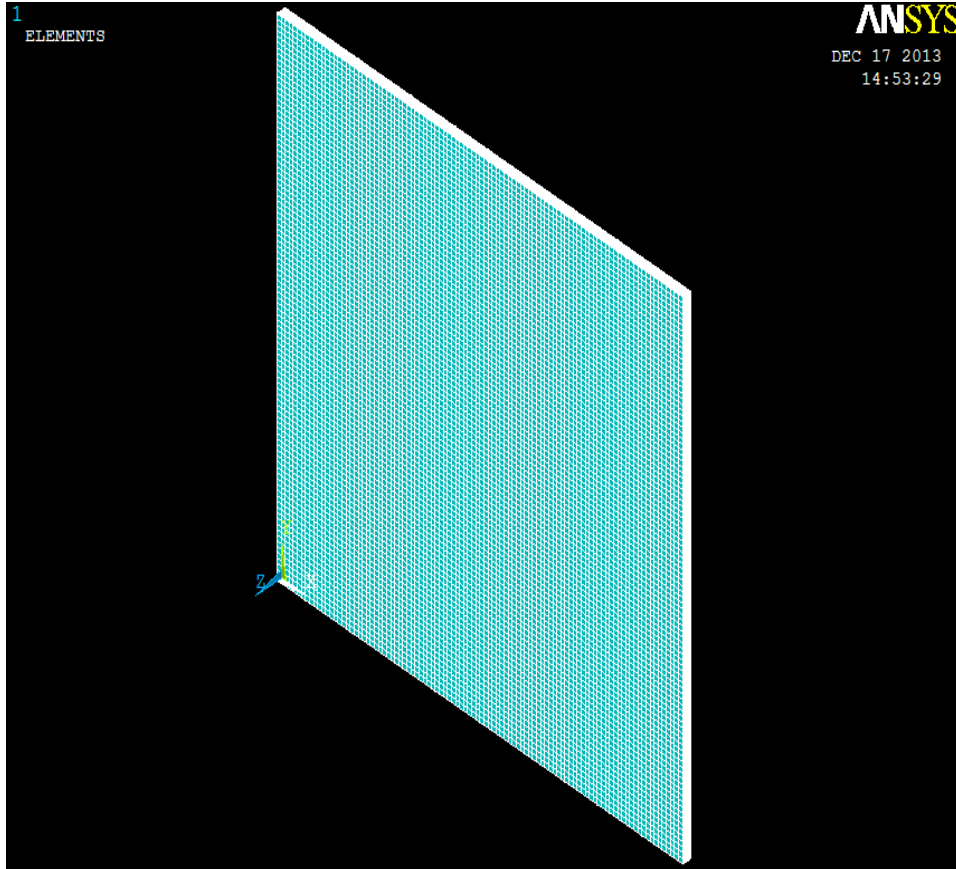


Fig. 2. Isometric view of meshed plate

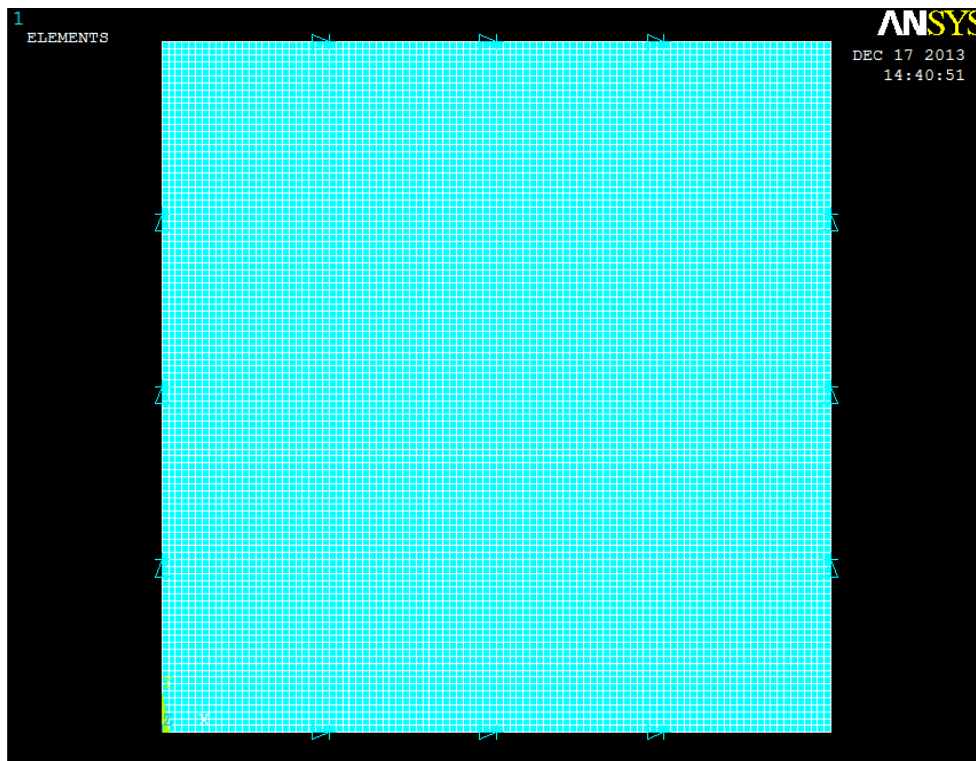


Fig.3: Simply supported plate

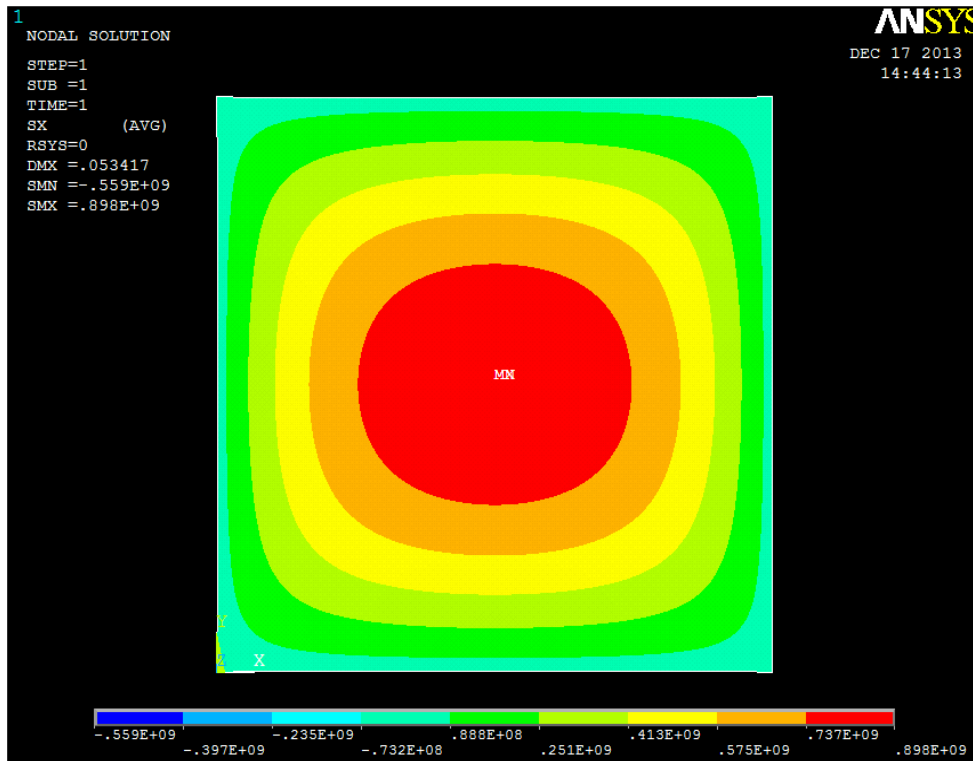


Fig.4: Stress

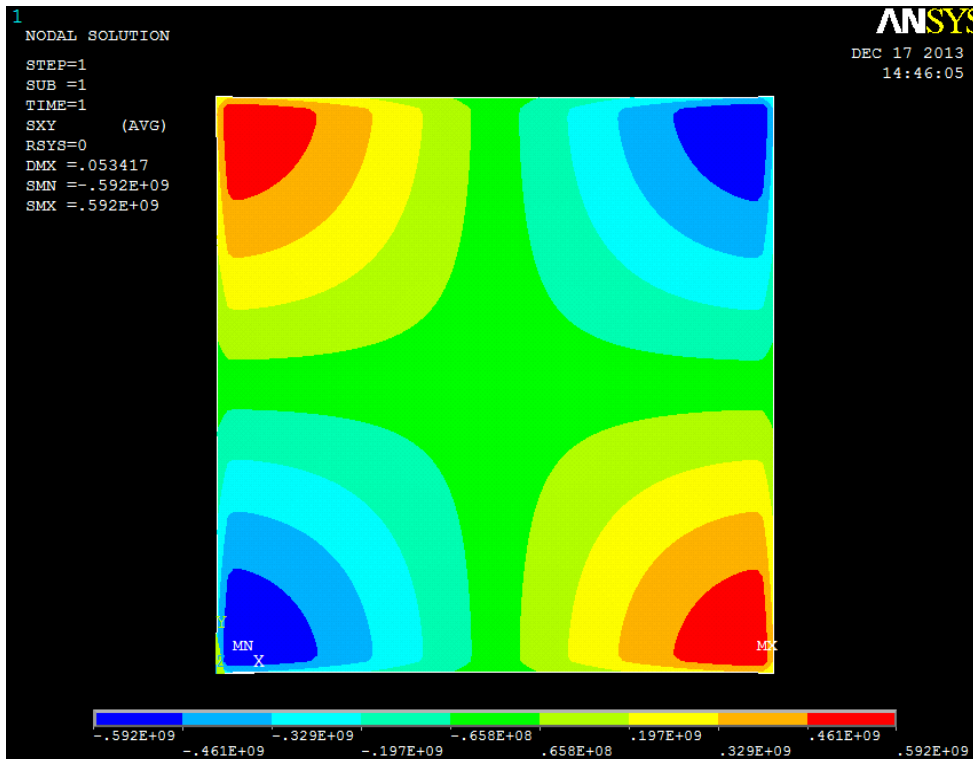


Fig.5 Shear stress

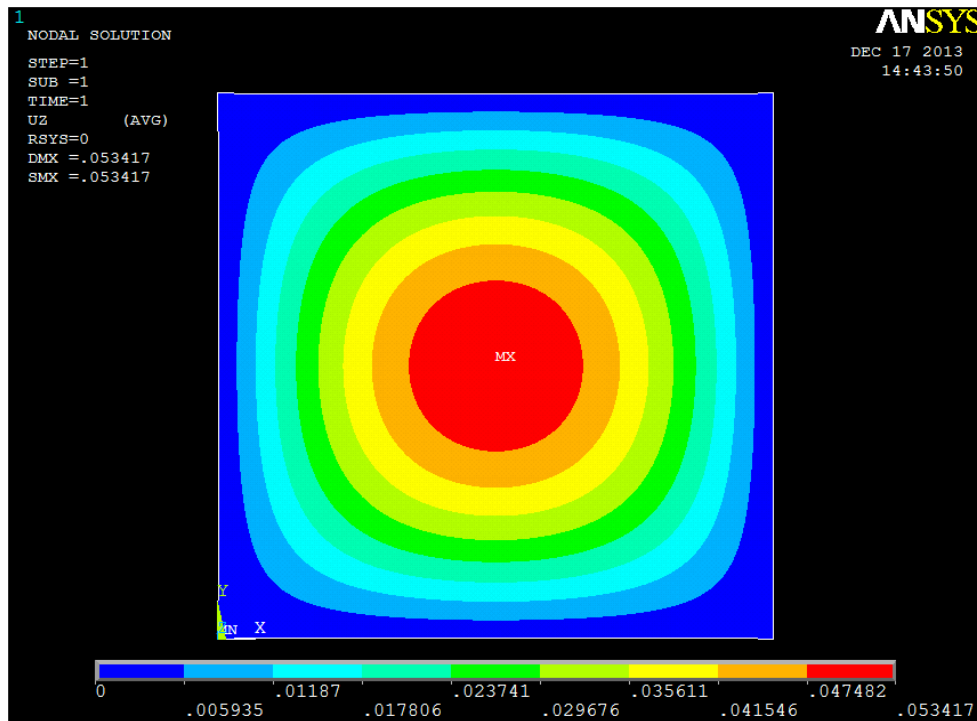


Fig.6. Deflection

4. Results

The aspect ratio of the plate is varied and under constant thermomechanical load condition and deflection, stress and shear stress are determined. The FGM plate is considered to be simply supported and the effect of values of volume fraction exponent 'n' i.e. pure ceramic plate ($n=0$), pure metal plate ($n=\infty$) and FGM plate ($n=1$ and 2) following Power law-FGM are studied. The results are presented graphically in terms of non-dimensional parameters i.e. non-dimensional deflection (\bar{u}_z), non-dimensional stress ($\bar{\sigma}_x$), non-dimensional shear stress ($\bar{\sigma}_{xy}$), strain (e_x) and shear strain (e_{xy}).

The non-dimensional parameters are defined as given below:

- (i) Non-dimensional deflection (\bar{u}_z) = u_z/h
- (ii) Non-dimensional Tensile stress ($\bar{\sigma}_x$) = σ_x/m
- (iii) Non-dimensional Shear stress ($\bar{\sigma}_{xy}$) = σ_{xy}/m

Where

- u_z = Deflection at the geometric center of the plate,
 σ_x = Tensile stress at the geometric center of the

plate,

σ_{xy} = Shear stress at the geometric center of the plate,

h = Plate thickness,

m = Unit pressure intensity ($=10 \times 10^5$ Pa)

4.1 Non-Dimensional Deflection (\bar{u}_z)

Fig.7 shows the effect of variation of aspect ratio (a/b) on non-dimensional deflection (\bar{u}_z) for simply supported FGM plate under udl in constant thermal environment for P-FGM. The result are compared for various volume fraction exponents 'n' in P-FGM.

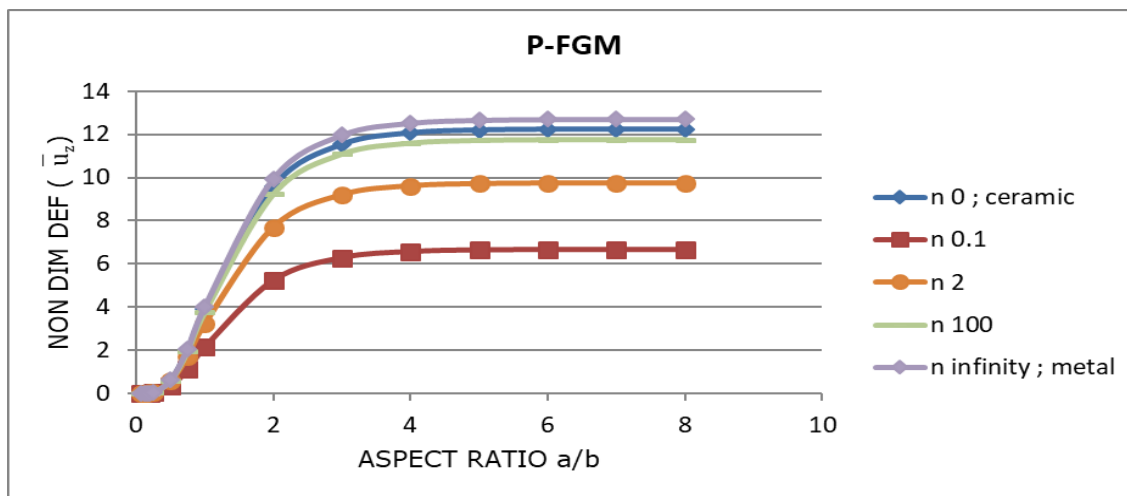


Fig.7: Effect of aspect ratio (a/b) on non-dimensional deflection (\bar{u}_z)

The graphs reveals the following information:

- It is evident that upto aspect ratio 4, the deflection increases and thereafter its value is almost constant.
- In case of metal plate (i.e. $n=\infty$) the deflection is maximum ($\bar{u}_z = 12.8$) while in case of ceramic plate, its value is about 12.1. These results are quite obvious, as metal is more sensitive to temperature than ceramic.
- The non-dimensional deflection values of FGM plates (i.e. $0 < n < \infty$) are much lower than that of metal plate. This clearly shows that the FGM plate can resist high temperature conditions.

(d) The non-dimensional deflection in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 1 (square plate), in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional deflection is approximately 2.1 whereas in metal rich region (P-FGM- $n=100$), it is 3.76.

4.2 Non-Dimensional Stress ($\bar{\sigma}_x$)

Fig.8 shows the effect of variation of aspect ratio (a/b) on non-dimensional stress ($\bar{\sigma}_x$) for simply supported FGM plate under udl in constant thermal environment for P-FGM. The results are compared for various volume fraction exponents (n) in P-FGM.

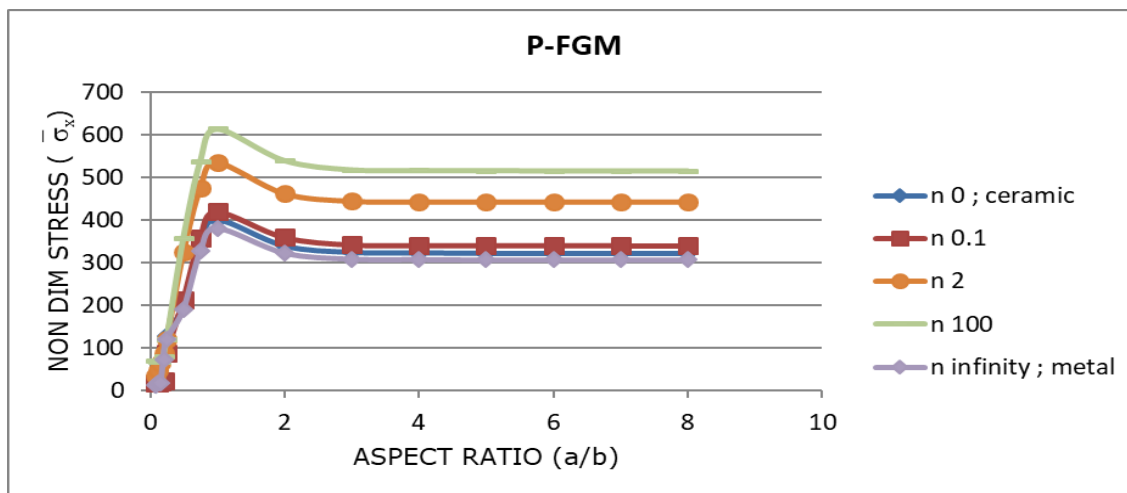


Fig.8: Effect of aspect ratio (a/b) on non-dimensional tensile stress ($\bar{\sigma}_x$)

The following observations are made while studying the effect of aspect ratio on stress:

- The non-dimensional stress reaches a maximum value for aspect ratio 1. It shows that the maximum non-dimensional stress occurs for square plate. Its peak value is about 613 in P-FGM ($n=100$).
- As the aspect ratio increase beyond 1 the non-dimensional stress reduces upto aspect ratio 2. Beyond the aspect ratio 2, non-dimensional stress becomes constant.
- The non-dimensional stress in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 2 (square plate), in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional stress is approximately 418 whereas in metal rich region (P-FGM- $n=100$), it is 613.

4.3 Non-dimensional Shear Stress ($\overline{\sigma_{xy}}$)

Fig.9 shows the effect of variation of aspect ratio (a/b) non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported FGM plate under udl in constant thermal environment for P-FGM. The results are compared for various volume fraction exponents (n) in P-FGM. The following observations are made while studying the effect of aspect ratio on shear stress:

(a) Initially upto aspect ratio equals to 2, the non-dimensional shear stress ($\overline{\sigma_{xy}}$) increases rapidly and thereafter its value is almost constant.

(b) In case of pure metallic plate (i.e. $n=\infty$), for square plate, the non-dimensional shear stress ($\overline{\sigma_{xy}}$) is minimum ($\overline{\sigma_{xy}} = 517$) while in case of pure ceramic plate, its value is about 544 which is near to the pure metal plate and it is lower than that of FGM of different configuration ($0 < n < \infty$). The product of coefficient of thermal expansion and modulus of elasticity for both metal and ceramic are nearly equal that and hence the shear stress of graded plate is not found to be intermediate to metal and ceramic plate.

(c) The non-dimensional shear stress in the ceramic rich region is found to be lower than metal rich region. For example at aspect ratio 2, in ceramic rich region (P-FGM- $n=0.1$) the non-dimensional shear stress is 749 whereas in metal rich region (P-FGM- $n=100$), it is 927. This shows that FGM plate can resist high temperature conditions.

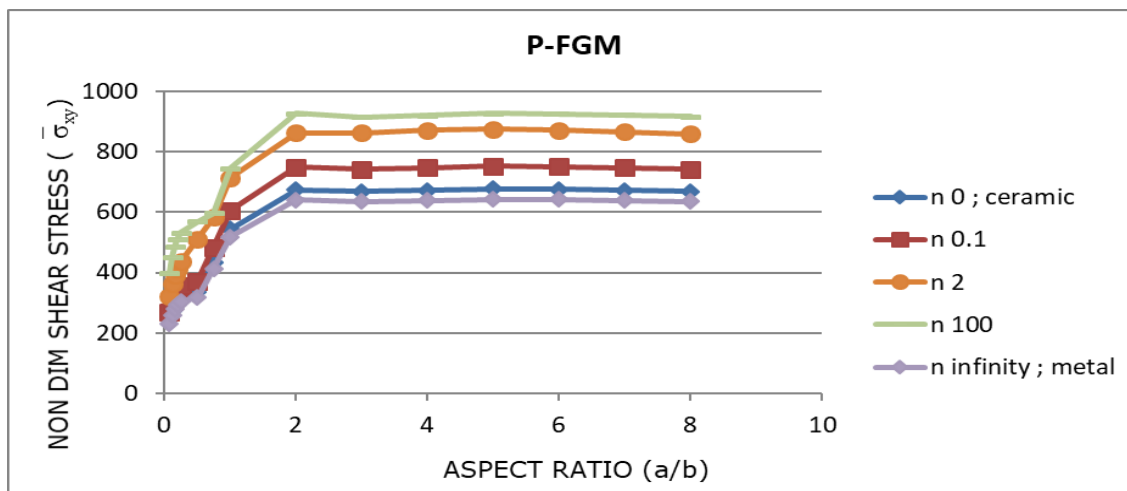


Fig.9: Effect of aspect ratio (a/b) on non-dimensional shear stress ($\overline{\sigma_{xy}}$)

5. Conclusion

The behaviour of FGM plate under thermo-mechanical environment was studied. The work includes parametric study performed by varying volume fraction distribution and aspect ratio. The following important conclusions may be drawn:

- (a) The deflection values of FGM plates (i.e. $0 < n < \infty$) are lower than that of metal plate. This clearly shows that the FGM plate can resist high temperature conditions. At lower temperature rise for the various volume fraction exponents, the deflections are closer to each other but as we increase the temperature, the deflection for the various volume fraction exponents diverge.
- (b) The non-dimensional shear stress diverges as the value of volume fraction exponent increases. As the value of volume fraction exponent 'n' is increased i.e. approaching towards pure metal, the magnitude of non-dimensional shear stress increased.

FG plates provide a high ability to withstand thermal stresses, which reflects its ability to operate at elevated temperatures. The FGMs provide a highly stable response for the thermal loading comparing to that of the isotropic materials.

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